Influence of Dephasing on Shot Noise in an Electronic Mach-Zehnder Interferometer

Florian Marquardt and C. Bruder
Departement Physik und Astronomie, Universität Basel, Klingelbergstrasse 82, 4056 Basel, Switzerland
(Received 19 June 2003; published 6 February 2004)

We analyze shot noise under the influence of dephasing in an electronic Mach-Zehnder interferometer, of the type that was realized recently [Yang Ji *et al.*, Nature (London) **422**, 415 (2003)]. Using a model of dephasing by a fluctuating classical field, we show how the usual partition noise expression $\mathcal{T}(1-\mathcal{T})$ is modified. We study the dependence on the power spectrum of the field, which is impossible in simpler approaches such as the dephasing terminal, against which we compare. We remark on shot noise as a tool to distinguish thermal smearing from genuine dephasing.

DOI: 10.1103/PhysRevLett.92.056805 PACS numbers: 73.23.–b, 03.65.Yz, 72.70.+m

Quantum interference effects form an important part of mesoscopic physics. Therefore, it is important to understand how interference is suppressed by the action of a fluctuating environment (such as phonons or other electrons), a phenomenon known as dephasing (or decoherence). This has been the subject of many recent experimental studies [1–7].

However, often the "visibility" of the interference pattern can also be diminished by phase averaging, when electrons with a spread of wavelengths contribute to the current, or when some parameter fluctuates slowly. Recently, a remarkable interference experiment has been performed using a Mach-Zehnder setup fabricated from the edge channels of a two-dimensional electron gas in the integer quantum Hall effect regime [8]. Besides measuring the current as a function of the phase difference between the paths, the authors also measured the shot noise to distinguish between phase averaging and "real" dephasing. While both effects suppress the interference term in the current, they may affect differently the partition noise which is nonlinear in the transmission probability [9]. The idea of using shot noise to learn more about dephasing is promising, connecting two fundamental issues in mesoscopic physics.

Most theoretical works on dephasing in mesoscopic physics are concerned with its influence on the average current only (see Refs. [10–15] and references therein), although there have been a few studies of shot noise in this context [16]. In this Letter, we present the first analysis of shot noise for an electronic one-channel Mach-Zehnder interferometer under the influence of dephasing (Fig. 1). We consider dephasing produced by a fluctuating classical potential, which describes either true nonequilibrium radiation impinging on the system or the thermal part of the environmental noise. This approach has been employed quite often in the past [12,17], is exact in the first case, and should be a reliable approximation for $T \gg eV$ in the second case. In particular, we are interested in the influence of the power spectrum of the environmental fluctuations on the shot-noise result, a question that goes beyond the phenomenological dephasing terminal model [16,18–21].

Model and general results.—We consider noninteracting, spin-polarized electrons. By solving the Heisenberg equation of motion for the electron field $\hat{\Psi}$ moving at constant velocity v_F (linearized dispersion relation), under the action of a fluctuating potential $V(x,t)$ (without backscattering), we obtain

$$\hat{\Psi}(x,\tau) = \int \frac{dk}{\sqrt{2\pi}} e^{-i\epsilon_k\tau} \sum_{\alpha=1}^{3} t_\alpha(k,\tau)\hat{a}_\alpha(k) e^{s_\alpha i k_F x} \quad (1)$$

for the electron operator at the output terminal 3. We have $t_3 = 1$, $s_{1,2} = 1$, $s_3 = -1$, the reservoir operators obey $\langle \hat{a}^\dagger_\alpha(k)\hat{a}_\beta(k')\rangle = \delta_{\alpha\beta}\delta(k-k')f_\alpha(k)$, with f_α the distribution function in reservoir α, and the integration is over $k > 0$ only. The amplitudes t_1, t_2 for an electron to go

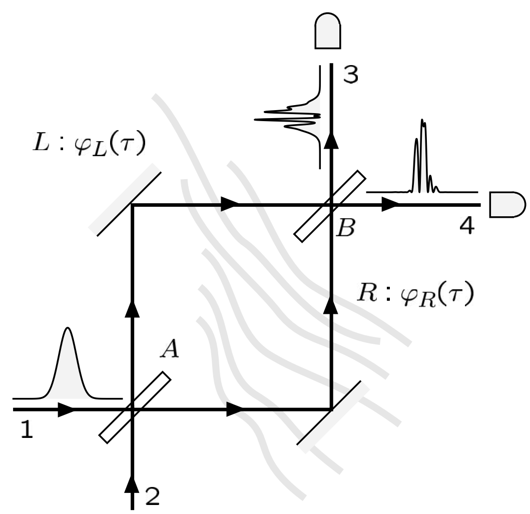

FIG. 1. The Mach-Zehnder interferometer setup analyzed in the text. In the case shown here, the fluctuations of the environment are fast compared with the temporal extent of the wave packet (determined by temperature or voltage, see text). The probability density of the incoming wave packet and its two outgoing parts is shown.

 0031-9007/04/92(5)/056805(4)$22.50

from terminal 1 or 2 to the output terminal 3 are time dependent:

$$t_1(k,\tau) = t_A t_B e^{i\varphi_R(\tau)} + r_A r_B e^{i\varphi_L(\tau)} e^{i(\phi + k\delta x)} \qquad (2)$$

$$t_2(k,\tau) = t_A r_B e^{i\varphi_L(\tau)} e^{i(\phi + k\delta x)} + r_A t_B e^{i\varphi_R(\tau)} \qquad (3)$$

Here $t_{A/B}$ and $r_{A/B}$ are energy-independent transmission and reflection amplitudes at the two beam splitters ($t_j^* r_j = -t_j r_j^*$), δx is a possible path-length difference, and ϕ is the Aharonov-Bohm phase due to the enclosed magnetic flux. The electron accumulates random phases while moving along the left or right arm: $\varphi_{L,R}(\tau) = -\int_{-\tau_{L,R}}^{0} dt' \, V(x_{L,R}(t'), \tau + t')$, where τ is the time when the electron leaves the second beam splitter after traveling for a time $\tau_{L,R}$ along the path described by $x_{L,R}(t)$. Note that in our model the total traversal times $\tau_{L,R}$ enter only at this point, and we assumed the interaction to be confined to the interferometer region.

The output current following from (1) has to be averaged over the fluctuating phases; i.e., it depends on phase-averaged transmission probabilities $T_1 = |t_1|^2$ and $T_2 = 1 - T_1$:

$$\langle T_1 \rangle_\varphi = T_A T_B + R_A R_B + 2z(r_A r_B)^* t_A t_B \cos(\phi + k\delta x), \qquad (4)$$

The interference term is suppressed by $z \equiv \langle e^{i\delta\varphi} \rangle_\varphi$, where $\delta\varphi = \varphi_L - \varphi_R$[assuming $V(x,t)$ and thus $\delta\varphi$ symmetrically distributed around 0]. We have $z = \exp(-\langle \delta\varphi^2 \rangle / 2)$ for Gaussian $\delta\varphi$. This factor decreases the visibility of the interference pattern $I(\phi)$. However, such a suppression can also be brought about by the k integration, if $\delta x \neq 0$ (thermal smearing).

Our main goal is to calculate the shot-noise power S at zero frequency. It can be split into two parts:

$$S = \int d\tau \langle\langle \hat{I}(\tau)\hat{I}(0)\rangle\rangle_\varphi - \langle\langle \hat{I}(0)\rangle\rangle_\varphi^2 = \int d\tau \langle\langle \hat{I}(\tau)\rangle\langle \hat{I}(0)\rangle\rangle_\varphi - \langle\langle \hat{I}(0)\rangle\rangle_\varphi^2 + \int d\tau \langle\langle \hat{I}(\tau)\hat{I}(0)\rangle - \langle \hat{I}(\tau)\rangle\langle \hat{I}(0)\rangle\rangle_\varphi. \qquad (5)$$

The first integral on the right-hand side describes shot noise of a classical current $I(\tau) = \langle \hat{I}(\tau) \rangle$, due to the fluctuating conductance. We denote its noise power as $S_{\rm cl}$. It rises quadratically with $\langle \hat{I} \rangle$, as is known from $1/f$ noise in mesoscopic conductors [22].

The second integral is evaluated by inserting (1) and applying Wick's theorem (similar formulas appear in Ref. [23]):

$$\langle\langle \hat{I}(\tau)\hat{I}(0)\rangle - \langle \hat{I}(\tau)\rangle\langle \hat{I}(0)\rangle\rangle_\varphi = \left(\frac{ev_F}{2\pi}\right)^2 \int dk\, dk' \sum_{\alpha,\beta=1,2,3} f_\alpha(k)[1 - f_\beta(k')] K_{\alpha\beta}(\tau) e^{i(\epsilon_{k'} - \epsilon_k)\tau}. \qquad (6)$$

Here $K_{\alpha\beta}$ is a correlator of four amplitudes: We have $K_{33} = 1$, $K_{3\alpha} = K_{\alpha 3} = 0$, and

$$K_{\alpha\beta}(\tau) \equiv \langle t_\alpha^*(k,\tau) t_\beta(k',\tau) t_\alpha(k,0) t_\beta^*(k',0) \rangle_\varphi, \qquad (7)$$

for $\alpha, \beta = 1, 2$.

We note that the τ range of the oscillating exponential under the integral in (6) is determined by the Fermi functions, i.e., by voltage and temperature. This has to be compared with the correlation time τ_c of the environment [the typical decay time of the phase correlator $\langle \delta\varphi(\tau)\delta\varphi(0)\rangle$]. For $eV\tau_c \ll 1$ and $T\tau_c \ll 1$ ("fast environment"), the major contribution of the integration comes from $|\tau| \gg \tau_c$, where $K_{\alpha\beta}$ factorizes into

$$K_{\alpha\beta}(\tau) \approx K_{\alpha\beta}(\infty) \equiv |\langle t_\alpha^*(k,0) t_\beta(k',0)\rangle_\varphi|^2. \qquad (8)$$

This yields the noise power

$$\frac{S_{\rm fast}}{e^2 v_F/2\pi} = \int dk \sum_{\alpha,\beta=1,2} f_\alpha(1 - f_\beta) |\langle t_\alpha^* t_\beta \rangle_\varphi|^2 + f_3(1 - f_3), \qquad (9)$$

where we have set $f_{\alpha,\beta} = f_{\alpha,\beta}(k)$ and $t_{\alpha,\beta} = t_{\alpha,\beta}(k,0)$. We conclude that the shot noise for a "fast" environment is *not* given by an expression of the form $\langle \mathcal{T} \rangle_\varphi (1 - \langle \mathcal{T} \rangle_\varphi)$, which would be obtained from a simple classical model (see discussion below). Indeed, we have

$$|\langle t_1^* t_2 \rangle_\varphi|^2 - \langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi) = (z^2 - 1) R_B T_B. \qquad (10)$$

The remainder of the noise power from Eq. (6) [with $K_{\alpha\beta}(\tau) - K_{\alpha\beta}(\infty)$ inserted in Eq. (6)] will be denoted $S_{\rm fluct}$. It yields a contribution to the Nyquist noise $S_{V=0}$, but apart from that it becomes important only at larger V, T. With this definition, the full noise power can always be written as

$$S = S_{\rm fast} + S_{\rm fluct} + S_{\rm cl}. \qquad (11)$$

In the other limit, when the τ integration is dominated by $|\tau| \ll \tau_c$ ("slow environment"), we can use $K_{\alpha\beta}(\tau) \approx K_{\alpha\beta}(0)$, which yields the phase average of the usual shot-noise result:

$$\frac{S_{\rm slow}}{e^2 v_F/2\pi} = \int dk \langle (f_1 T_1 + f_2 T_2)[1 - (f_1 T_1 + f_2 T_2)] \rangle_\varphi + f_3(1 - f_3). \qquad (12)$$

Discussion.—The phase averages can be evaluated exactly if the potential $V(x,t)$ (and therefore $\delta\varphi$) is assumed to be a Gaussian random field of zero mean. In the following, we present explicit results for the case $T = 0$, $\delta x eV/v_F \ll 1$, where the visibility is decreased purely by dephasing. We need the following Fourier transforms ($\lambda = \pm$):

$$\hat{g}_\lambda(\omega) \equiv \int d\tau \, e^{i\omega\tau} [e^{\lambda \langle \delta\varphi(\tau)\delta\varphi(0)\rangle} - 1], \qquad (13)$$

$$I_\lambda(V) \equiv \int_0^{eV} d\omega \left(1 - \frac{\omega}{eV}\right) \hat{g}_\lambda(\omega). \qquad (14)$$

The shot noise becomes ($\tilde{\phi} = \phi + k_F \delta x$):

$$\frac{S - S_{V=0}}{e^3 V/2\pi} = \frac{eV}{\pi} z^2 R_A R_B T_A T_B [\cos(2\tilde{\phi})\hat{g}_-(0) + \hat{g}_+(0)] + |\langle t_1^* t_2 \rangle_\varphi|^2 + \frac{1}{\pi} z^2 R_B T_B \{-2\cos(2\tilde{\phi}) R_A T_A I_-(V) + (R_A^2 + T_A^2) I_+(V)\}. \quad (15)$$

The first line corresponds to $S_{\rm cl}$, the second to $S_{\rm fast}$, and the rest to $S_{\rm fluct} - S_{V=0}$. At $V \to 0$, the integrals $I_\pm(V)$ vanish and $S_{\rm fast}$ dominates. At large $eV\tau_c \gg 1$, we can use the sum rule $I_\lambda(V) \to \pi[z^{-2\lambda} - 1]$ and find the last three lines to combine to $\langle T_1(1-T_1)\rangle_\varphi$, i.e., $S_{\rm slow}$. The Nyquist noise is ϕ independent:

$$S_{V=0} = \frac{e^2}{2\pi^2} z^2 R_B T_B \int_0^\infty d\omega\, \omega \hat{g}_+(\omega). \quad (16)$$

The results are illustrated in Figs. 2 and 3, where $S(V)$ shows the crossover between the fast and "slow" limits. Although $S_{\rm fast}$ can vanish, the total current noise S remains finite, due to the Nyquist part. For the plots we assumed $T_A = 1/2$ and a Gaussian phase correlator, $\langle \delta\varphi(\tau)\delta\varphi(0)\rangle = \langle \delta\varphi^2\rangle \exp[-(\tau/\tau_c)^2]$. An application of the general theory presented here to specific situations includes calculating $\langle \delta\varphi(\tau)\delta\varphi(0)\rangle$, starting from the correlator $\langle VV\rangle_{q\omega}$ of the potential fluctuations $V(x,t)$ (cf. [12] for an example). The contribution of potential modes to $\langle \delta\varphi^2\rangle$ is suppressed for $|q| < 1/R$ (R: typical distance between the paths) and becomes maximal for small $|v_F q - \omega|$.

For the other limit of a large path-length difference $\delta x \gg v_F/eV$, or $\delta x \gg v_F/T$, the interference term is already suppressed completely due to wavelength averaging. Then $S_{\rm cl}$ vanishes, since $\langle \hat{I}(\tau)\rangle$ is independent of the fluctuating phase. In addition, we find (at $T = 0$)

$$\frac{S - S_{V=0}}{e^3 V/2\pi} = T_A R_A (T_B - R_B)^2 + z^2 T_B R_B (T_A^2 + R_A^2)\left[1 + \frac{I_+(V)}{\pi}\right]. \quad (17)$$

For a fast environment, we have $I_+(V) \to 0$, such that, for $T_A = 1/2$, Eq. (17) becomes $[(T_B - R_B)^2 + 2z^2 R_B T_B]/4$, which turns into $(T_B - R_B)^2/4$ for $z \to 0$. This could be distinguished from the k-averaging result, but it describes the case of large energy transfers, as opposed to "pure dephasing." On the other hand, in the limit of large voltages ("slow environment," $eV\tau_c \gg 1$), we have $I_+(V) \to \pi[z^{-2} - 1]$ and, for $T_A = 1/2$, Eq. (17) turns into $(T_B^2 + R_B^2)/4$, which is equal to the result obtained for pure k averaging alone.

We have pointed out already that even $S_{\rm fast}$ does not lead to the simple result $\langle T_1\rangle_\varphi(1 - \langle T_1\rangle_\varphi)$. However, the latter form does indeed apply if we consider injecting only a "narrow beam" of electrons into terminal 1 [i.e.,

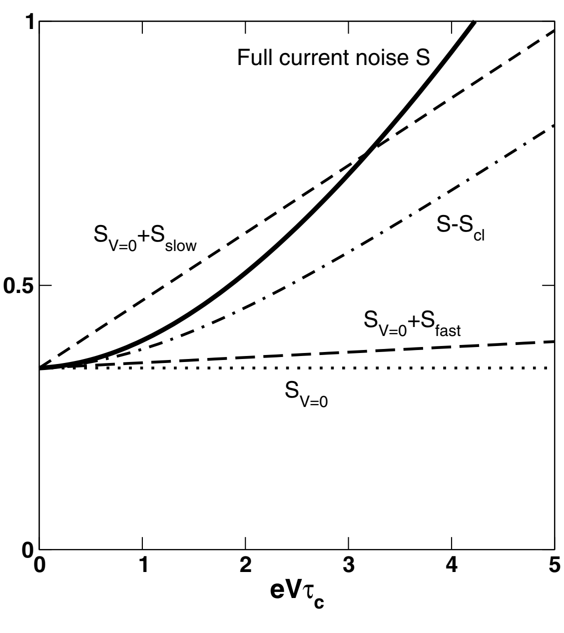

FIG. 2. Typical behavior of the full current noise S as a function of $eV\tau_c$. At higher voltages, the dependence on V is quadratic, due to $S_{\rm cl}$. When $S_{\rm cl}$ is subtracted, the slope at large $eV\tau_c$ is determined by $S_{\rm slow}$ (i.e., $\langle T_1(1-T_1)\rangle_\varphi$), while that at low voltages is always determined by $S_{\rm fast}$ (i.e., $|\langle t_1^* t_2\rangle_\varphi|^2$). Parameters: $T = 0$, $\delta x = 0$, $\phi = 0$, $T_A = 1/2$, $z = 1/e$, $T_B = 0.4$.

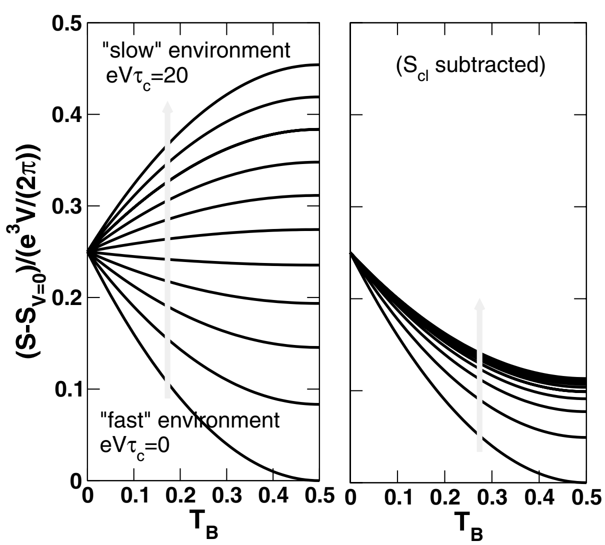

FIG. 3. Normalized shot noise $(S - S_{V=0})/(e^3V/2\pi)$ as a function of the transmission T_B of the second beam splitter for small visibility ($z = 1/e$). The different curves show the succession from a "fast" environment to a "slow" one (bottom to top: $eV\tau_c = 0, 2, 4, \ldots, 20$). In the right panel, the contribution from S_{cl} [first line of (15)] has been dropped, to demonstrate the convergence against the result for a slow bath, $\langle T_1(1-T_1)\rangle_\varphi$ (topmost curve). Other parameters are as in Fig. 2.

$f_\alpha(k) = 0$ except for $f_1(k) = 1$ within $[k_F, k_F + eV/v_F]$, which is not equivalent to the previous situation regarding shot noise (cf. [24] in this respect). We get the following for $eV\tau_c \ll 1$:

$$S - S_{\rm cl} = \frac{e^3 V}{2\pi} \langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi), \tag{18}$$

while the case $eV\tau_c \gg 1$ is described by $S_{\rm slow}$.

Comparison with other models.—We compare our results in the fully incoherent limit ($z = 0$) with two other models; namely, the dephasing terminal [16,20,21] and a simple model of a stream of regularly injected electrons [25] reaching the output port according to classical probabilities. We focus on zero temperature and the case $T_A = 1/2$. At small path-length difference $eV\delta x/v_F \ll 1$ (no k averaging), we obtain $\langle T_1 \rangle_\varphi (1 - \langle T_1 \rangle_\varphi) = 1/4$ both for the classical model and the narrow beam of electrons, $(T_B - R_B)^2/4$ for our shot-noise expression in the fast case, and $(T_B^2 + R_B^2)/4$ both for the slow case and from the dephasing terminal [26]. In the opposite limit of large δx only the result for the classical model changes, coinciding with the slow case $(T_B^2 + R_B^2)/4$ that also holds without dephasing. Thus, in this case a shot-noise measurement most likely will not be able to reveal the additional presence of dephasing. Concerning the experiment of Ref. [8], this could invalidate the conclusion drawn from the noise measurements (carried out at high voltages) if δx (whose precise value is unknown) were not small enough. Repeating the measurements at intermediate values of the visibility will yield more insights into these questions.

In conclusion, we analyzed the effects of dephasing on shot noise for an electronic Mach-Zehnder interferometer. We generalized the scattering theory of shot noise to include dephasing induced by fluctuations of a classical potential. This has enabled us to analyze the dependence of shot noise on the power spectrum of the fluctuations, going beyond simpler phenomenological approaches, to which we have compared our results. We have identified a crossover between two regimes, those of a fast and a slow environment. We have pointed out that a shot-noise measurement cannot reveal the presence of dephasing on top of thermal averaging, for environmental fluctuations slower than the inverse voltage or temperature. Our theory may be applied to other single-channel interferometer geometries as well, even in the presence of backscattering at the junctions.

We thank M. Heiblum for useful comments and for sending us a preprint of Ref. [8]. Useful discussions with W. Belzig, M. Büttiker, C. Egues, and E. Sukhorukov are gratefully acknowledged. This work was supported by the Swiss NSF and the NCCR nanoscience.

[1] P. Mohanty, E. M. Q. Jariwala, and R. A. Webb, Phys. Rev. Lett. **78**, 3366 (1997).
[2] E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and H. Shtrikman, Nature (London) **391**, 871 (1998).
[3] D. P. Pivin, A. Andresen, J. P. Bird, and D. K. Ferry, Phys. Rev. Lett. **82**, 4687 (1999).
[4] A. E. Hansen, A. Kristensen, S. Pedersen, C. B. Sørensen, and P. E. Lindelof, Phys. Rev. B **64**, 045327 (2001).
[5] D. Natelson, R. L. Willett, K. W. West, and L. N. Pfeiffer, Phys. Rev. Lett. **86**, 1821 (2001).
[6] K. Kobayashi, H. Aikawa, S. Katsumoto, and Y. Iye, J. Phys. Soc. Jpn. **71**, L2094 (2002).
[7] F. Pierre and N. O. Birge, Phys. Rev. Lett. **89**, 206804 (2002).
[8] Y. Ji, Y. Chung, D. Sprinzak, M. Heiblum, D. Mahalu, and H. Shtrikman, Nature (London) **422**, 415 (2003).
[9] V. A. Khlus, J. Exp. Theor. Phys. **66**, 1243 (1987); G. B. Lesovik, JETP Lett. **49**, 592 (1989).
[10] I. L. Aleiner, N. S. Wingreen, and Y. Meir, Phys. Rev. Lett. **79**, 3740 (1997).
[11] Y. Levinson, Europhys. Lett. **39**, 299 (1997).
[12] G. Seelig and M. Büttiker, Phys. Rev. B **64**, 245313 (2001).
[13] F. Marquardt and C. Bruder, Phys. Rev. B **65**, 125315 (2002).
[14] F. Marquardt and C. Bruder, Phys. Rev. B **68**, 195305 (2003).
[15] G. Seelig, S. Pilgram, A. N. Jordan, and M. Büttiker, Phys. Rev. B **68**, 161310 (2003).
[16] J. H. Davies, J. C. Egues, and J. W. Wilkins, Phys. Rev. B **52**, 11 259 (1995); C. Texier and M. Büttiker, Phys. Rev. B **62**, 7454 (2000).
[17] B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky, J. Phys. C **15**, 7367 (1982); S. Chakravarty and A. Schmid, Phys. Rep. **140**, 195 (1986); A. Stern, Y. Aharonov, and Y. Imry, Phys. Rev. A **41**, 3436 (1990).
[18] M. Büttiker, Phys. Rev. B **33**, 3020 (1986).
[19] M. J. M. de Jong and C. W. J. Beenakker, in *Mesoscopic Electron Transport*, edited by L. P. Kouwenhoven, L. L. Sohn, and G. Schön, NATO ASI Series Vol. 345 (Kluwer Academic, Dordrecht, 1997).
[20] M. J. M. de Jong and C. W. J. Beenakker, Physica (Amsterdam) **230A**, 219 (1996).
[21] S. A. van Langen and M. Büttiker, Phys. Rev. B **56**, R1680 (1997).
[22] Sh. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University Press, Cambridge, England, 1996).
[23] G. B. Lesovik, JETP Lett. **70**, 208 (1999).
[24] U. Gavish, Y. Levinson, and Y. Imry, Phys. Rev. Lett. **87**, 216807 (2001).
[25] Ya. M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
[26] There are indications that the ansatz used for calculating shot noise in the dephasing terminal approach underestimates shot noise in the limit of small δx (see [27]).
[27] F. Marquardt and C. Bruder (to be published).