

Toolbox of resonant quantum gates in Circuit QED

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We propose the implementation of fast resonant gates in circuit quantum electrodynamics for quantum information processing. We show how a suitable utilization of three-level superconducting qubits inside a resonator constitutes a key tool to perform a wide range of two-qubit resonant gates, improving the operation speed when compared to dispersive techniques. Furthermore, we propose the implementation of a two-dimensional cluster state in an array of $N \times N$ qubits for the sake of one-way quantum computation, where the generation time grows linearly with N .

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Circuit quantum electrodynamics (QED) is a novel field combining atomic physics and quantum optical cavity QED concepts with superconducting circuits [1, 2, 3]. Its fundamental dynamics is understood within the Jaynes-Cummings model, describing the interaction between a two-level system and a single field mode [4]. In Circuit QED, superconducting qubits are considered as artificial atoms [5, 6] interacting with on-chip one-dimensional resonators playing the role of cavities [7]. These mesoscopic devices are candidates for implementations in quantum information processing [8] due to their inherent tunability and scalability properties, at least as good as trapped ions [9] or quantum dots [10].

To implement quantum algorithms in circuit QED [11] within the standard quantum computing approach, the sequential realization of fast high-fidelity quantum gates is required [12]. Several implementations of quantum operations in coupled superconducting qubits have been proposed [13] and the implementation of a controlled-NOT gate has been achieved [14]. Furthermore, coherent coupling [15] and quantum information exchange [16] of two superconducting qubits through a cavity bus and cavity state synthesis [17] have been realized. Scalable architectures in circuit QED may require efficient designs of two-dimensional cavity arrays [18]. These allow scalable standard quantum computation, and also (as we will show) the realization of two-dimensional cluster states for one-way quantum computing [19, 21]. To speed up operations and, thus, beat decoherence, it would be desirable to employ resonant gates in circuit QED architectures. Resonant gates, based on first-order couplings between qubits and cavities, are much faster than the commonly used dispersive gates, based on second-order couplings.

In this Letter, we propose the implementation of a toolbox of resonant quantum gates, improving on speed and fidelity when compared to dispersive gates. To this end, a

key concept will be the use of an auxiliary excited state for the qubits and the use of the cavity as a resonant mediator of the qubit interactions. We will explain how to realize, resonantly and efficiently, paradigmatic two-qubit gates, as controlled-phase (CPHASE), iSWAP, and Bogoliubov gates [22]. Some of the latter, together with suitable one-qubit gates, can form universal sets for quantum computation. We test our results with full numerical simulations involving decoherence mechanisms. As a first nontrivial application, we propose the realization of a cluster state for one-way quantum computing [19] in a two-dimensional array of cavities in circuit QED [18].

We consider two three-level superconducting qubits, 1 and 2, with $|g_{1,2}\rangle$, $|e_{1,2}\rangle$, and $|a_{1,2}\rangle$ being their first three lower energy levels, respectively. The usefulness of auxiliary qubit levels in the superconducting qubit context is currently manifest for phase qubit operations [20, 23], for circuit QED quantum optics applications [24], as well as for a recent demonstration of gates in circuit QED [11]. The qubits are coupled to a coplanar waveguide cavity and their dynamics is described, after a rotating-wave approximation, by the Hamiltonian

$$\begin{aligned}
 H = & \sum_{l=g,e,a;q=1,2} E_{l_q} |l_q\rangle \langle l_q| + \hbar\omega_r a^\dagger a \\
 & + \hbar g_{g_1 e_1} (\sigma_{g_1 e_1}^+ a + \sigma_{g_1 e_1}^- a^\dagger) \\
 & + \hbar g_{e_1 a_1} (\sigma_{e_1 a_1}^+ a + \sigma_{e_1 a_1}^- a^\dagger) \\
 & + \hbar g_{g_2 e_2} (\sigma_{g_2 e_2}^+ a + \sigma_{g_2 e_2}^- a^\dagger) \\
 & + \hbar g_{e_2 a_2} (\sigma_{e_2 a_2}^+ a + \sigma_{e_2 a_2}^- a^\dagger). \quad (1)
 \end{aligned}$$

Here $\sigma_{k,l}^+ \equiv |l\rangle\langle k|$, $\sigma_{kl}^- \equiv |k\rangle\langle l|$, while a (a^\dagger) are the bosonic annihilation (creation) operators of the resonator field mode, E_{l_q} is the energy for level l of qubit q , ω_r is the cavity mode frequency, and the g 's denote the vacuum Rabi coupling strengths. We assume that the qubit levels are anharmonic and that each transition can be

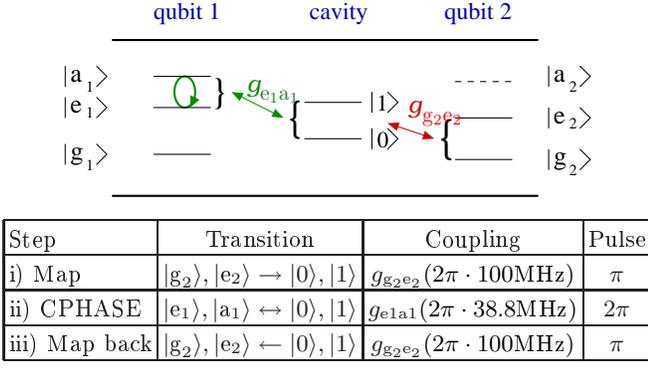


FIG. 1: Sketch and protocol for the resonant implementation of a CPHASE gate between qubits 1 and 2. The coupling values are the ones used for the simulation presented in Fig.2.

tuned to match the cavity frequency, e.g. by using AC-Stark-shift fields [25], thereby effectively switching on the qubit-cavity couplings for the respective transition. Conversely, we may consider the use of a tunable cavity [26].

We now show how to implement a CPHASE gate between the two qubits in a resonant manner. We assume that at a certain point of a quantum computation the tripartite system (qubits+cavity) is in the following state

$$|\Psi_i\rangle = (\alpha |g_1g_2\rangle + \beta |g_1e_2\rangle + \gamma |e_1g_2\rangle + \delta |e_1e_2\rangle) \otimes |0\rangle, \quad (2)$$

where α, β, γ , and δ are arbitrary complex amplitudes and $|0\rangle$ is the cavity vacuum. A CPHASE gate is implemented on qubits 1 and 2 after the protocol displayed in Fig. 1 is applied. In step (i), the state of qubit 2, encoded in its two low energy levels, $|g_2\rangle$ and $|e_2\rangle$, is mapped onto a photonic cavity qubit through a resonant coupling between qubit 2 and the cavity mode, while the first qubit remains off-resonant. It is possible to achieve this mapping without introducing extra phase factors, such that the state at that stage is $|\Psi\rangle = (\alpha |g_10\rangle + \beta |g_11\rangle + \gamma |e_10\rangle + \delta |e_11\rangle) \otimes |g_2\rangle$. In step (ii) [27], a 2π resonant pulse between the two upper energy levels of qubit 1, $|e_1\rangle$ and $|a_1\rangle$, and the cavity mode realizes a CPHASE gate among them, yielding $|\Psi\rangle = (\alpha |g_10\rangle + \beta |g_11\rangle + \gamma |e_10\rangle - \delta |e_11\rangle) \otimes |g_2\rangle$. In the last step (iii), the cavity qubit is mapped back to qubit 2, thus implementing a CPHASE gate between qubits 1 and 2, leaving the decoupled cavity in the vacuum state

$$|\Psi_f\rangle = (\alpha |g_1g_2\rangle + \beta |g_1e_2\rangle + \gamma |e_1g_2\rangle - \delta |e_1e_2\rangle) \otimes |0\rangle. \quad (3)$$

To prove the feasibility of the proposed resonant protocol for implementing a CPHASE gate, we have employed a time-dependent Lindblad master equation simulation, taking into account cavity losses and qubit relaxation and dephasing. We have considered a charge qubit where the ratio between the charge and the Josephson energies, E_J/E_C , can be tuned from the charge to the transmon

regime [28]. Even if the transmon regime enjoys the best decoherence parameters, it is not obvious that this limit will be optimal for having the required level anharmonicity when tuning different qubit transitions to the cavity mode, or vice versa. Note that, in contrast to the case of dispersive gates, where only virtual photons are involved, in step (ii) we populate resonantly the cavity with a real photon during a finite operation time. The simulations indicate that the anharmonicity condition for the qubit levels and the decay of cavity photons do not prevent the generation of a fast high-fidelity CPHASE gate between the qubits. As an initial state, we have chosen

$$|\bar{\Psi}_i\rangle = \frac{1}{\sqrt{2}} (|g_1g_2\rangle + |e_1e_2\rangle) \otimes |0\rangle, \quad (4)$$

where the maximally entangled qubits should be a sensitive probe of decoherence processes. Figure 2 shows the reduced density operator of the qubits at the initial and final state, after the three steps to achieve the CPHASE gate. The values we considered in the simulation are $\kappa_{\text{cav}} = 10$ kHz for the cavity decay rate (with a cavity frequency of $\omega_{\text{cav}} = 2\pi \cdot 5.5\text{GHz}$), and $T_1 = 7.3\mu\text{s}$ and $T_2 = 0.5\mu\text{s}$ for the time-scales of qubit relaxation and decoherence, respectively. The coupling strengths have been obtained from diagonalizing the charge qubit Hamiltonian for $E_J/E_C = 2.53$ and are indicated in the table in Fig.1. In the simulation, level positions have been shifted during the time-evolution, assuming an appropriate AC Stark shift, to realize the required qubit-cavity resonances. The density operators are expressed in the computational basis $\{|g_1g_2\rangle, |g_1e_2\rangle, |e_1g_2\rangle, |e_1e_2\rangle\}$. We find a final state fidelity of 98.5 % in our simulation, where the fidelity is defined as $F = \text{Tr}(|\sqrt{\rho_{\text{lossy}}}\rho_{\text{ideal}}\sqrt{\rho_{\text{lossy}}}|)$, with ρ_{ideal} and ρ_{lossy} being the final state density operators for the ideal operation and for the real evolution, respectively.

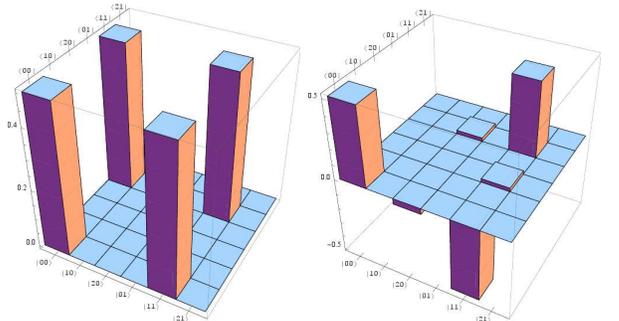


FIG. 2: Density operator of the initial state Ψ_i (with $\alpha = \delta = 1, \beta = \gamma = 0$) and of the final state after the implementation of the CPHASE gate, in a simulation involving qubit relaxation and dephasing, and cavity decay; expressed in the computational basis of the two qubits.

We consider now a resonant protocol to generate the two-qubit iSWAP gate, a paradigmatic gate appearing in

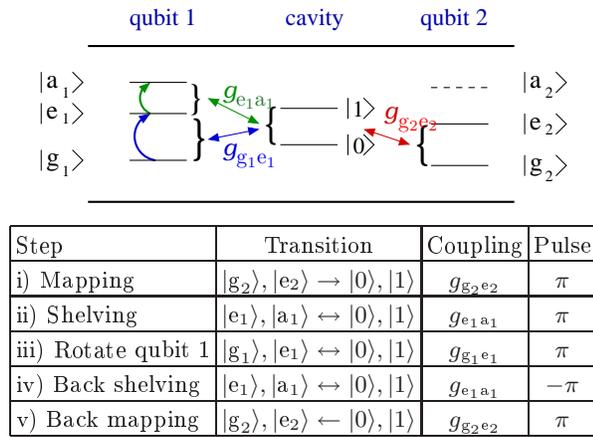


FIG. 3: Sketch and protocol for the resonant implementation of an iSWAP gate between qubits 1 and 2.

Step	Transition	Coupling	Pulse
i) Mapping	$ g_2\rangle, e_2\rangle \rightarrow 0\rangle, 1\rangle$	$g_{g_2 e_2}$	π
ii) Rotate qubit 1	$ g_1\rangle \leftrightarrow e_1\rangle$	Ω_R	π
iii) Shelving	$ e_1\rangle, a_1\rangle \leftrightarrow 0\rangle, 1\rangle$	$g_{e_1 a_1}$	π
iv) Entanglement	$ g_1\rangle, e_1\rangle \leftrightarrow 0\rangle, 1\rangle$	$g_{g_1 e_1}$	2θ
v) Back shelving	$ e_1\rangle, a_1\rangle \leftrightarrow 0\rangle, 1\rangle$	$g_{e_1 a_1}$	$-\pi$
vi) Back rotation	$ g_1\rangle \leftrightarrow e_1\rangle$	Ω_R	π
vii) Back mapping	$ e_2\rangle, g_2\rangle \leftarrow 0\rangle, 1\rangle$	$g_{g_2 e_2}$	π

FIG. 4: Protocol for the resonant implementation of a Bogoliubov gate between qubits 1 and 2. Ω_R is the frequency of an external classical field resonant to the transition frequency between qubit states $|g_1\rangle$ and $|e_1\rangle$.

circuit QED, where a dispersive qubit-cavity-qubit coupling is usually considered. This standard approach to generate the iSWAP gate is based on a weak and slow second-order coupling. The resonant steps to implement an iSWAP gate in circuit QED are shown in Fig. 3, showing the advantages of the proposed resonant techniques.

Most two-qubit gates form a universal set if properly accompanied by the suitable one-qubit gate. In this sense, it should be enough to have a two-qubit gate, say the CPHASE, that can be done fast and efficiently. However, depending on the quantum computation one wants to implement, a more diverse gate toolbox leads to a more efficient protocol. We show now that the proposed resonant tools are not only valid for generating resonantly and efficiently the CPHASE or the iSWAP gate, but can also produce some other exotic two-qubit gates. We exemplify this with the Bogoliubov gate, see Fig. 4, appearing in the context of simulations of the dynamical evolution of spin models with quantum gates [22].

It is also desirable that the accompanying one-qubit gates could be implemented with fast resonant pulses. As is known, rotations around the x and y axis can be implemented by sending a resonant driving field to the

transition frequency of the corresponding qubit. Rotations around the z axis correspond to a detuning, which for charge qubits can be implemented via changing the Josephson energy E_J or by AC Stark shifts. In particular, this allows to implement the Hadamard gate, either by a 180 degrees rotation around $x+z$ or by the sequence $H = e^{i\pi/2} R_x(\pi) R_y(\pi/2)$, where $R_i(\theta)$ is a rotation by θ around axis i . Together with CPHASE (or CNOT) and a $\pi/8$ gate, this forms a universal set of gates for quantum computing. The fast gates available in the resonant toolbox of one- and two-qubit gates increase the number of operations realizable within the given coherence time.

As a direct application of the gates proposed here, we will show their use for the realization of a multiqubit entangled state of N^2 qubits, the so-called cluster state.

One-way quantum computing [19] is an alternative model to the standard approach based on quantum gates. It requires the initial generation of a two-dimensional cluster state, containing all necessary entanglement for the local and sequential implementation of a quantum algorithm. A cluster state can be obtained by the action of a CPHASE gate between neighbouring qubits, assuming all of them initially in state $(|g\rangle + |e\rangle)/\sqrt{2}$. Its experimental realization is a key element when trying to implement one-way quantum computing in different physical systems. We present here a resonant protocol for realizing a cluster state of N^2 qubits in two dimensions, considering a two-dimensional cavity grid [18] as the most suitable architecture for achieving this goal. This architecture consists of a 2D square array of one-dimensional resonators where the qubits are placed at the crossings of the cavities. We assume that the qubits are initially in their ground states and the cavities contain zero photons.

To generate the initial state, one can send an external field resonant to the transition frequency of all qubits to implement a y-rotation for a $\pi/2$ pulse. These local qubit rotations can be done simultaneously so that the corresponding operation time is $\tau_y = \frac{\pi}{2\Omega_R}$. The qubits will now be entangled by performing a CPHASE gate between each qubit and their nearest neighbours. Although all CPHASE gates mathematically commute and could be implemented simultaneously [19], the proposed architecture of the cavity grid imposes critical constraints. In particular, one CPHASE gate can be implemented at most in one cavity at the same time. A solution to optimize the number of steps is shown in Fig. 5. It consists in implementing simultaneously all CPHASE gates on the first row using the vertical cavities, plus the ones we can do on the first column with the unused horizontal cavities. We repeat this step N times by shifting the row to the top and the column to the right. Then, we will have implemented all required CPHASE gates between neighbouring qubits except the ones at the crossings between the horizontal and vertical cavities during the process. Finally, we implement the missing gates on the two diagonals in two steps due to the architectural constraints.

The total time used to realize the 2D cluster state is

$$\begin{aligned} \tau_{\text{total}} &= \frac{\pi}{2\Omega_R} + N \left(\frac{\pi}{g_{g_2 e_2}} + \frac{2\pi}{g_{e_1 a_1}} + \frac{\pi}{g_{g_2 e_2}} \right) + \\ & 2 \left(\frac{\pi}{g_{g_2 e_2}} + \frac{2\pi}{g_{e_1 a_1}} + \frac{\pi}{g_{g_2 e_2}} \right) \\ &= \frac{\pi}{2\Omega_R} + (N+2) \cdot 2\pi \cdot \left(\frac{g_{g_2 e_2} + g_{e_1 a_1}}{g_{g_2 e_2} g_{e_1 a_1}} \right), \quad (5) \end{aligned}$$

where the second and third term on the r.h.s. of the first line corresponds to the simultaneously done and the last missing CPHASE gates, respectively. The total time scales with N , whereas the number of qubits is N^2 , producing a resonant and efficient technique to build 2D cluster states in the cavity grid architecture.

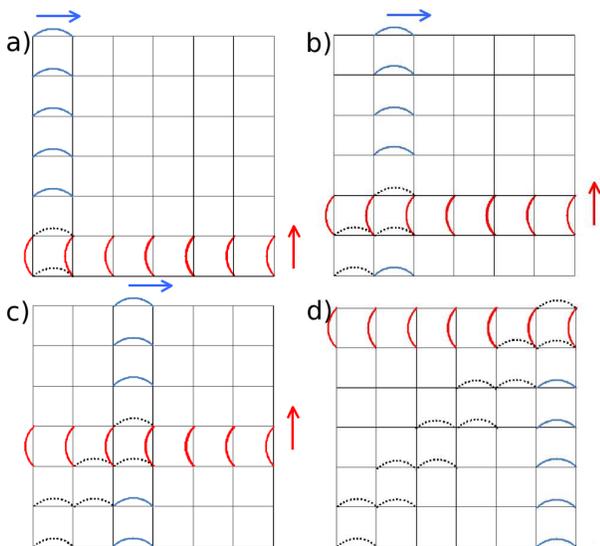


FIG. 5: (Color online). Generation of a cluster state in a 2D cavity grid by implementing simultaneous CPHASE gates following the steps (a-d). The grid is composed of $2N$ cavities and contains N^2 qubits, one at each crossing point. The performed CPHASE gates for the lines are displayed in red and for the columns in blue. The row is shifted to the top and the column to the right. The missing gates at the crossings are indicated as dashed black lines.

We have shown that it is possible to build a circuit QED toolbox of resonant and high-fidelity one-qubit and two-qubit gates for the sake of implementing standard quantum computing. We have shown numerically that one can obtain fast resonant CPHASE gates of high fidelity by using an auxiliary qubit level and the cavity photon to establish qubit communication. Furthermore, we have introduced a method of generating efficiently a cluster state in a two-dimensional architecture for imple-

mentating one-way quantum computing. We expect that the introduced toolbox of resonant and efficient gates will enhance the theoretical and experimental research in quantum information applications in circuit QED.

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