Intracavity Squeezing Can Enhance Quantum-Limited Optomechanical Position Detection through Deamplification

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It has been predicted and experimentally demonstrated that by injecting squeezed light into an optomechanical device, it is possible to enhance the precision of a position measurement. Here, we present a fundamentally different approach where the squeezing is created directly inside the cavity by a nonlinear medium. Counterintuitively, the enhancement of the signal-to-noise ratio works by deamplifying precisely the quadrature that is sensitive to the mechanical motion without losing quantum information. This enhancement works for systems with a weak optomechanical coupling and/or strong mechanical damping. This can allow for larger mechanical bandwidth of quantum-limited detectors based on optomechanical devices. Our approach can be straightforwardly extended to quantum nondemolition qubit detection.

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Recent progress in cavity optomechanics [1,2] has been so exceptional that the precision of a position measurement has been pushed to the limit set by the principles of quantum mechanics, the so-called standard quantum limit (SQL) [3–5]. A measurement precision close to the SQL has been demonstrated in optomechanical devices with cavities both in the optical [6–8] and in the microwave [9] domain. Optomechanical position detection is not only of fundamental interest but finds also application in acceleration [10,11], magnetic field [12,13], and force detectors [14,15]. Thus, an important goal for the future is to develop new techniques to enhance its precision on different optomechanical platforms. Seminal efforts have focused on gravitational wave detection in optomechanical interferometers [16–20]. The standard route to enhance the detection precision consists in injecting squeezed light into the interferometer [16,17,21]. This technique has recently been demonstrated in the Laser Interferometer Gravitational Wave Observatory (LIGO) [22] and in a cavity optomechanics setup [23]. Externally generated squeezed light could also find application in quantum nondemolition (QND) qubit state detection [24,25]. Injection losses are a major hindrance of the effectiveness of externally generated squeezed light. This has motivated a number of proposals aiming to create the squeezing directly inside the cavity. Intracavity squeezing could be generated by a Kerr medium [18,20,21,26–28] by the dissipative optomechanical interaction [29–31] in a multimode optomechanical system [32] or, potentially, by exploiting the ponderomotive squeezing [33–35].

In this Letter, we propose a new pathway to precision enhancement in optomechanical detection. In our approach, a nonlinear cavity is operated as a phase-sensitive parametric amplifier, as shown in Fig. 1. It amplifies a seed laser beam and its intensity fluctuations. Simultaneously, it *deamplifies* the phase quadrature where the mechanical vibrations are imprinted. At first sight, it might appear counterintuitive that deamplification can improve a (quantum) measurement. Here, we suggest that it is worth deamplifying a signal if the noise is suppressed by a larger factor, thus, obtaining a net enhancement of the signal-to-noise ratio. Indeed, our analysis shows that for optomechanical position detection, a deamplification of the phase quadrature induces only a limited suppression of the signal but simultaneously can strongly suppress the measurement noise. Our scheme could

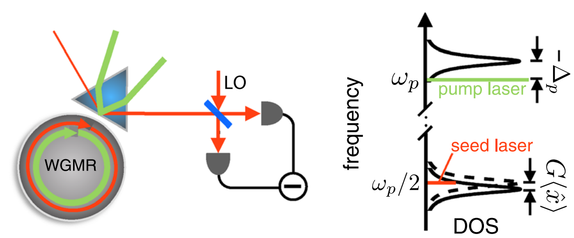

FIG. 1 (color online). Setup for parametrically amplified optomechanical position measurement. A whispering gallery mode resonator (WGMR) with a $\chi^{(2)}$ nonlinearity is operated as a degenerate parametric amplifier. The pump laser (frequency ω_p) drives the signal mode at its parametric resonance but is detuned compared to the pump mode resonance. An additional laser beam with the appropriate phase and frequency $\omega_p/2$ (seed laser) is amplified in the WGMR. The signal mode resonance depends on the amplitude $\langle\hat{x}\rangle$ of a mechanical mode deformation. Thus, the mechanical vibrations are imprinted in the seed laser phase shift detected in a homodyne setup.

be implemented using a crystalline whispering gallery mode resonator [36]. Such devices offer a well-established platform for optomechanics [1,2,37,38]. Resonators with an optical $\chi^{(2)}$ nonlinearity can be operated as parametric amplifiers in the quantum regime [39,40]. The exciting perspective of an interplay of optical and optomechanical nonlinearities has already inspired several theoretical investigations [41–44]. Alternative implementations of our scheme include optomechanical crystals [45] when made out of nonlinear materials such as AlN [46] and a Josephson parametric amplifier [47] coupled to a mechanical membrane or a qubit.

We consider a degenerate parametric amplifier (DPA) described by the standard linearized Hamiltonian [48]

$$H = i\hbar\bar{n}_p^{1/2}\nu(\hat{a}_s^\dagger\hat{a}_s^\dagger - \hat{a}_s\hat{a}_s)/2.$$

The signal mode (annihilation operator $\hat{a}_s$ and decay rate κ_s) is driven parametrically at its parametric resonance via an auxiliary pump mode, $\bar{n}_p$ denotes the number of photons circulating in the pump mode, and ν is the single-photon $\chi^{(2)}$ nonlinearity. The parametric amplifier is characterized by its pump parameter σ,

$$\sigma^2 = \bar{n}_p/\bar{n}_p^{(\mathrm{thr})}, \qquad \bar{n}_p^{(\mathrm{thr})} = \left(\frac{\kappa_s}{2\nu}\right)^2. \tag{1}$$

The signal mode reaches the threshold of self-sustained (optical parametric) oscillations when the photon number circulating in the pump mode equals $\bar{n}_p^{(\mathrm{thr})}$, corresponding to the pump parameter $\sigma = 1$. Below threshold, the cavity behaves as a phase-sensitive amplifier as discussed above.

We want to measure the displacement $\hat{x}$ of a mechanical resonator with eigenfrequency Ω, effective mass m, and decay rate Γ. The mechanical resonance could be internal to the optical resonator (e.g., a breathing mode) or refer to the vibrations of an external nano-object coupled evanescently. A displacement $\hat{x}$ induces a shift $-G\hat{x}$ of the signal mode frequency described by a Hamiltonian $\hat{H}_{\mathrm{OM}} = -\hbar G\hat{a}_s^\dagger\hat{a}_s\hat{x}$ [1]. We measure the displacement $\hat{x}$ by extracting the output signal phase of a seed drive injected at the cavity resonance, where it is most sensitive to the jittering of the optical resonance induced by the mechanical vibrations. When the seed laser injects a large number $\bar{n}_s$ of circulating photons into the signal mode (below, we specify this condition more precisely), we can linearize the optomechanical interaction [1]. Then the mechanical vibrations couple to the optical field quadrature $\hat{X} = (\hat{a}_s + \hat{a}_s^\dagger - 2\sqrt{n_s})/\sqrt{2}$ describing the amplitude fluctuations: $\hat{H}_{\mathrm{OM}} = -\hbar G\sqrt{2\bar{n}_s}\hat{X}\,\hat{x}$. We arrive at the Langevin equations for the optical signal mode quadratures $\hat{X}$ (amplitude) and $\hat{Y}$ (phase):

$$\dot{\hat{X}} = -(1-\sigma)\kappa_s\hat{X}/2 + \sqrt{\kappa_s}\hat{X}^{(\mathrm{in})},$$
$$\dot{\hat{Y}} = -(1+\sigma)\kappa_s\hat{Y}/2 + \sqrt{2\bar{n}_s}G\hat{x} + \sqrt{\kappa_s}\hat{Y}^{(\mathrm{in})}, \tag{2}$$

where $\hat{Y} = i(\hat{a}_s^\dagger - \hat{a}_s)/\sqrt{2}$, and $\hat{X}^{(\mathrm{in})}$ and $\hat{Y}^{(\mathrm{in})}$ are the standard vacuum input fields (the quantum fluctuations of the laser beam at the input) [48]. Here, we neglect intrinsic losses and the coupling to the pump mode fluctuations. We go beyond this ideal description below. As seen in Eq. (2), the presence of the nonlinear medium and the pump drive manifests itself in the deamplification of the phase quadrature and a corresponding amplification of the amplitude quadrature. In the limit $\sigma \to 0$, we recover the Langevin equations for a cavity measuring the mechanical displacement in the standard approach without squeezing.

To improve a measurement by deamplification might not sound promising. The measurement noise will be deamplified, but one could reasonably expect that this effect will be offset by the deamplification of the signal. Indeed, it is true that the response of the cavity to both the vacuum noise and the mechanical vibrations is decreased by the same factor. From Eq. (2), the intracavity phase quadrature in frequency space is

$$\hat{Y}[\omega] = \chi_Y(\omega)(\sqrt{2\bar{n}_s}G\hat{x}[\omega] + \sqrt{\kappa_s}\hat{Y}^{(\mathrm{in})}[\omega]) \tag{3}$$

with the intracavity susceptibility $\chi_Y = [-i\omega + (1+\sigma)\kappa_s/2]^{-1}$. We note in passing that the largest possible suppression, a factor of 2, occurs in the limit $\omega \to 0$ and $\sigma \to 1$. This is the well-known 3 dB limit of intracavity squeezing [49]. However, the suppression of the background noise and of the mechanical signal is different outside the cavity. From the input-output relation $\hat{Y}^{(\mathrm{out})} = \hat{Y}^{(\mathrm{in})} - \sqrt{\kappa_s}\hat{Y}$, we find

$$\hat{Y}^{(\mathrm{out})}[\omega] = [1 - \kappa_s\chi_Y(\omega)]\hat{Y}^{(\mathrm{in})}[\omega] - \sqrt{2\kappa_s\bar{n}_s}G\chi_Y(\omega)\hat{x}[\omega]. \tag{4}$$

From this formula, we see that the response of the phase quadrature of the transmitted signal to the mechanical vibrations is still governed by the intracavity susceptibility and is, thus, subject to the 3 dB limit of squeezing. In contrast, the output phase noise is squeezed below the 3 dB limit by the destructive interference between the reflected input noise and the response of the cavity to that noise. Indeed, it is well known that the output noise squeezing can be arbitrarily large [21]. Thus, we expect an overall enhancement of the measurement precision accompanied by deamplification. This behavior is displayed by the symmetrized spectral density of the output phase quadrature

$$\bar{S}_{YY}(\omega) = \int_{-\infty}^{\infty}\frac{dt}{2}e^{i\omega t}\langle\{\hat{Y}^{(\mathrm{out})}(t), \hat{Y}^{(\mathrm{out})}(0)\}\rangle, \tag{5}$$

i.e., the quantity measured in the homodyne setup; see Fig. 2.

We briefly comment on the similarities between our proposal and other schemes where interference effects enhance the optomechanical position detection. In dissipative optomechanical setups [29,30], the interference between the light impinging on the cavity and the light filtered by the cavity enhances the signal intensity rather

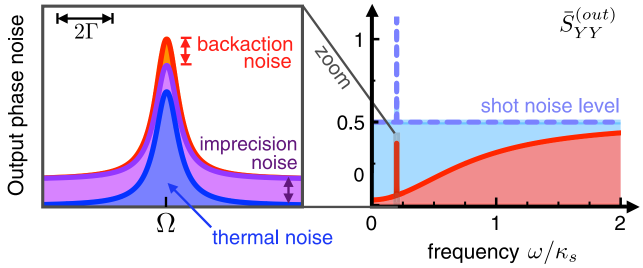

FIG. 2 (color online). Output phase noise $\bar{S}_{YY}^{(\mathrm{out})}$ as a function of frequency. Comparison between the phase noise in the presence and in the absence of the pump drive for the same number of circulating photons $\bar{n}_s$. In the presence of the pump laser (pump parameter $\sigma = 0.6$), the background noise inside the amplifier bandwidth is squeezed below the shot noise level by more than 3 dB. The signal amplitude is also reduced, but in this case, the reduction is bounded by the 3 dB limit. The number of circulating photons $\bar{n}_s$ is chosen to yield the minimum added noise allowed by the SQL, for $\sigma = 0.6$. Thus, the imprecision noise and the backaction noise (shown in the zoom) have the same intensity at the mechanical resonator eigenfrequency Ω. The remaining parameters are $\Omega = 0.2\kappa_s$, $\Gamma = 10^{-3}\kappa_s$, $k_B T/\hbar\Omega = 1$.

than reducing the measurement noise. Closer to our settings, a nonlinear cavity operated close to its static bistability [18,20,21,26–28] is formally equivalent to an effectively detuned DPA [27]. Because of the effective detuning, the amplitude and phase fluctuations become correlated. An important consequence of such correlations is that the SQL is reached only away from the mechanical resonance (whereas for us, it is reached precisely at resonance) [27]. For such a quantum-limited measurement, one should measure the quadrature whose homodyne signal is amplified by the cavity [27]. If one does not aim for a quantum-limited measurement, one can also measure the deamplified quadrature [20]. This leads to an improvement of the signal-to-noise ratio similar to the one observed here. However, this is accompanied by a loss of quantum efficiency because most of the information regarding the mechanical vibrations is imprinted on the other amplified quadrature, and, thus, the SQL would not be reached. Most important, there is not such a trade-off in our scheme where all information is imprinted on the deamplified quadrature.

In order to quantify the net enhancement of the measurement precision, it is convenient to define the measured noise referred back to the input $\bar{S}_{xx}^{(\mathrm{meas})} = \bar{S}_{YY}/(2\kappa_s\bar{n}_s G^2|\chi_Y|^2)$. Then, from Eq. (4) the measured noise takes the form $\bar{S}_{xx}^{(\mathrm{meas})}(\omega) = \bar{S}_{xx}(\omega) + \bar{S}_{xx}^{(\mathrm{add})}$ where $\bar{S}_{xx}(\omega)$ describes the symmetrized mechanical noise in the absence of optomechanical backaction, whereas $\bar{S}_{xx}^{(\mathrm{add})}$ is the noise added during the measurement. We are interested in the noise at frequency Ω where the mechanical spectrum is peaked. Since there is a typical number of circulating photons (specific of the device) that can be tolerated without inducing strong heating effects, we use as a figure of merit of our measurement scheme the added noise $\bar{S}_{xx}^{(\mathrm{add})}(\Omega)$ for a fixed effective number of circulating photons, $\bar{n} = \bar{n}_s + \bar{n}_p/\eta$. The device-dependent coefficient η reflects the different impact on heating of signal and pump photons. In first approximation, one can identify η with the ratio of the energy quanta of the signal and pump modes, $\eta \approx 1/2$. Below, we show that internally generated optical squeezing can strongly enhance the precision when

$$\mathcal{C}_{\mathrm{thr}} \equiv \frac{g_0^2\kappa_s}{\eta\Gamma\nu^2} = \frac{4g_0^2\bar{n}_p^{(\mathrm{thr})}}{\eta\Gamma\kappa_s} \ll 1. \tag{6}$$

Here, $g_0 = G/x^{\mathrm{ZPF}}$ is the optical frequency shift when the oscillator is displaced by the quantum length scale $x^{\mathrm{ZPF}} = \sqrt{\hbar/m\Omega}$ [1]. The parameter $\mathcal{C}_{\mathrm{thr}}$, which we refer to as *threshold cooperativity,* is the optomechanical cooperativity if $\bar{n}_s = \bar{n}_p^{(\mathrm{thr})}/\eta$ photons were in the signal mode. It quantifies the ratio of optomechanical and nonlinear coupling. Notice that in the absence of squeezing, the SQL is reached for the optomechanical cooperativity $\mathcal{C} = 1/4$ [1]. Thus, if $\mathcal{C}_{\mathrm{thr}} \ll 1$, it is not possible to achieve a precision close to the SQL by injecting all available photons $\bar{n}_s \sim \bar{n}_p^{(\mathrm{thr})}/\eta$ directly into the signal mode. Instead, one can enhance the measurement precision by injecting part of the photons into the pump mode to generate squeezing, as shown below.

We now calculate the added noise $\bar{S}_{xx}^{(\mathrm{add})}$. We distinguish between two different contributions [3–5]: the so-called *imprecision noise* $\bar{S}_{xx}^{(\mathrm{imp})}(\omega)$ and *the backaction noise* $\bar{S}_{xx}^{(\mathrm{back})}(\omega)$. The former is due to the shot noise phase fluctuations. The latter is the additional mechanical noise induced by the backaction of the light onto the mechanics. It can be expressed as $\bar{S}_{xx}^{(\mathrm{back})}(\omega) = |\chi_M(\omega)|^2\bar{S}_{FF}(\omega)$ in terms of the mechanical susceptibility $\chi_M(\omega) = m^{-1}(\Omega^2 - \omega^2 + i\omega\Gamma)^{-1}$ and the noise spectrum $\bar{S}_{FF}$ of the radiation pressure force $\hat{F} = \sqrt{2\bar{n}_s}\hbar G\hat{X}$. We note in passing that our measurement scheme could also find application in the detection of any degree of freedom coupled dispersively to the cavity, e.g., a qubit [4]. From Eq. (2), we can readily derive the identity $\bar{S}_{xx}^{(\mathrm{imp})}(\omega)\bar{S}_{FF}(\omega) = \hbar^2/4$ valid for all values of σ. It is well known that when this equality holds, both the position detection of resonant vibrations and the QND qubit state detection are quantum limited [3–5]. We compute the overall added noise $\bar{S}_{xx}^{(\mathrm{add})} = \bar{S}_{xx}^{(\mathrm{imp})} + \bar{S}_{xx}^{(\mathrm{back})}$ from Eq. (2),

$$\frac{\bar{S}_{xx}^{(\mathrm{imp})}}{\bar{S}_{xx}^{\mathrm{SQL}}} = \frac{(1-\sigma)^2 + 4\Omega^2/\kappa_s^2}{8\mathcal{C}_{\mathrm{thr}}[\eta\bar{n}/n_p^{(\mathrm{thr})} - \sigma^2]}, \qquad \frac{\bar{S}_{xx}^{(\mathrm{back})}}{\bar{S}_{xx}^{\mathrm{SQL}}} = \frac{\bar{S}_{xx}^{\mathrm{SQL}}}{4\bar{S}_{xx}^{(\mathrm{imp})}}. \tag{7}$$

Here, we have introduced the minimum added noise allowed by the SQL $\bar{S}_{xx}^{\mathrm{SQL}} = \hbar/m\Omega\Gamma$ [3–5]. Figure 3(a) shows the added noise Eq. (7) as a function of the circulating photon number $\bar{n}$ and the pump parameter σ. For $\sigma = 0$ (no

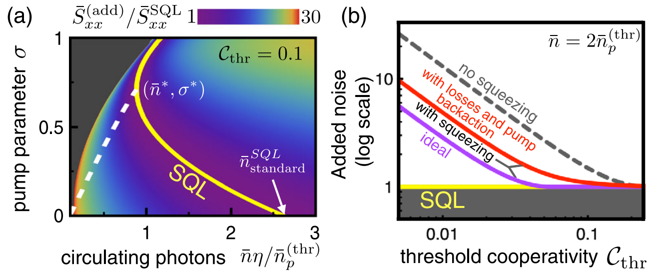

FIG. 3 (color online). (a) Added noise at the mechanical frequency as a function of the total number of circulating photons $\bar{n}=\bar{n}_s+\bar{n}_p/\eta$ and the pump parameter σ, for $\mathcal{C}_{\mathrm{thr}}=0.1$. The coordinates $(\eta\bar{n}/\bar{n}_p^{(\mathrm{thr})},\sigma)$ where the added noise equals the SQL are indicated by the yellow solid line; see Eq. (8). For $\bar{n}<\bar{n}^*$, the added noise is always larger than the SQL. In this case, the minimum noise for a fixed $\bar{n}$ is realized on the white dashed line, where the pump parameter $\tilde{\sigma}$ is given by Eq. (11). (b) Added noise as a function of the cooperativity $\mathcal{C}_{\mathrm{thr}}$, for $\bar{n}=\bar{n}_p^{(\mathrm{thr})}/\eta$: in the ideal case with squeezing ($\kappa^{(\mathrm{abs})}=0$, $|\Delta_p|/\kappa_p\to\infty$), in the presence of losses, pump backaction, and squeezing ($\kappa^{(\mathrm{abs})}=\Omega^2/\kappa_s$, $-\Delta_p/\kappa_p\to 5$), and in the absence of squeezing and losses ($\sigma=0$). In both panels, we have chosen $\kappa_s/\Omega=10$. In the right panel, we have chosen $\eta=1/2$, $\kappa_s=\kappa_p$, and equal couplings of the mechanics to the pump and the signal modes.

circulating photons in the pump mode), we recover the result for standard optomechanical detection. The SQL is reached for $\bar{S}_{xx}^{(\mathrm{add})}=2\bar{S}_{xx}^{(\mathrm{imp})}=2\bar{S}_{xx}^{(\mathrm{back})}=\bar{S}_{xx}^{\mathrm{SQL}}$ [3–5]; see, also, the zoom in Fig. 2. From Eq. (7), we find the required photon number

$$\bar{n}^{\mathrm{SQL}}(\sigma)=\frac{\bar{n}_p^{(\mathrm{thr})}}{\eta}\left[\frac{(1-\sigma)^2+4\Omega^2/\kappa_s^2}{4\mathcal{C}_{\mathrm{thr}}}+\sigma^2\right]. \tag{8}$$

It is shown as a yellow solid line in Fig. 3(a). By minimizing $\bar{n}^{\mathrm{SQL}}(\sigma)$ as a function of σ, we find the minimal number of circulating photons $\bar{n}^*$ necessary to reach the SQL and the corresponding optimal pump parameter σ^*,

$$\bar{n}^*=\bar{n}^{\mathrm{SQL}}(\sigma^*),\qquad \sigma^*=(1+4\mathcal{C}_{\mathrm{thr}})^{-1}. \tag{9}$$

Compared to the standard scheme, where the SQL is reached for $\bar{n}^{\mathrm{SQL}}_{\mathrm{standard}}=\bar{n}^{\mathrm{SQL}}(\sigma=0)$ circulating photons, the required number of photons is suppressed by a factor of

$$\bar{n}^{\mathrm{SQL}}_{\mathrm{standard}}/\bar{n}^*=\frac{1+4\Omega^2/\kappa_s^2}{1-(4\mathcal{C}_{\mathrm{thr}}+1)^{-1}+4\Omega^2/\kappa_s^2}. \tag{10}$$

The suppression factor increases monotonically with increasing optical nonlinearity (decreasing threshold cooperativity $\mathcal{C}_{\mathrm{thr}}$) and reaches the asymptotic value $\kappa_s^2/4\Omega^2$ for large optical nonlinearities ($\mathcal{C}_{\mathrm{thr}}\ll 1$) in the bad-cavity limit $\Omega\ll\kappa_s$. Our method is still useful even when it is not possible to reach the SQL because the typical number of circulating photons tolerated in the device is too small (smaller than $\bar{n}^*$). In this case, the added noise remains larger than $\bar{S}_{xx}^{\mathrm{SQL}}$, yet it can still be decreased by the squeezing. By minimizing $\bar{S}_{xx}^{(\mathrm{add})}$ in Eq. (7) as a function of σ for a fixed $\bar{n}$ (smaller than $\bar{n}^*$), we find the optimal pump parameter

$$\tilde{\sigma}=\frac{\mathcal{B}}{2}-\left(\frac{\mathcal{B}^2}{4}-\frac{\eta\bar{n}}{\bar{n}_p^{(\mathrm{thr})}}\right)^{1/2},\qquad \mathcal{B}=1+\frac{\eta\bar{n}}{\bar{n}_p^{(\mathrm{thr})}}+4\frac{\Omega^2}{\kappa_s^2}. \tag{11}$$

It increases monotonically with the number of circulating photons and reaches the value $\tilde{\sigma}=\sigma^*$ for $\bar{n}=\bar{n}^*$; see the white dashed line in Fig. 3(a).

Next, we go beyond the simple description of an ideal parametric amplifier taking into account intrinsic losses, the radiation pressure coupling between the mechanics and the pump mode, and that signal photons can be up-converted by the $\chi^{(2)}$ interaction. In this scenario, the vibrations are imprinted also in the pump output. Thus, the backaction of the pump field gives rise to additional mechanical noise. Moreover, the up-converted photons can decay via the pump mode decreasing the measurement precision. A similar increase of the measurement imprecision occurs in the presence of material absorption. A full analysis of these effects is provided in the Supplemental Material [50] and can be summarized as follows: (i) the additional backaction is a small fraction of the minimal added noise $\bar{S}_{xx}^{\mathrm{SQL}}$ for $\mathcal{C}_{\mathrm{thr}}\ll 1$ (the regime of interest in this work); (ii) the imprecision noise is substantially increased only for loss rates $\kappa_s^{(\mathrm{loss})}$, $\kappa_s^{(\mathrm{loss})}\gtrsim\Omega^2/\kappa_s$. The overall loss rate takes the form

$$\kappa_s^{(\mathrm{loss})}=\kappa_s^{(\mathrm{abs})}+4\nu^2\bar{n}_s\kappa_p/(4\Delta_p^2+\kappa_p^2), \tag{12}$$

where $\kappa_s^{(\mathrm{abs})}$ is the rate of material absorption, while κ_p and Δ_p are the pump mode decay rate and detuning, respectively. Thus, for a typical signal photon number $\bar{n}_s\sim n_p^{(\mathrm{thr})}$, a detuning of a few linewidths is enough to suppress the additional decay via the pump mode. This allows a deamplification-induced enhancement of the measurement precision in a broad range of threshold cooperativities $\mathcal{C}_{\mathrm{thr}}$; see Fig. 3(b).

The inevitable increase of the intensity fluctuations in the proposed measurement scheme represents a potential contradiction of the assumption of small fluctuations inherent to the linearized Langevin equations (2). However, it can be shown that the enhanced fluctuations remain compatible with the linearization, provided that the single-photon nonlinearity ν is not too large, $\nu\ll\Omega$ [50].

The regime of small threshold cooperativities $\mathcal{C}_{\mathrm{thr}}\ll 1$ is realized in state-of-the-art lithium-niobate microdisks [39,40,51]. These devices have breathing modes with eigenfrequencies Ω in the MHz range. Typical single-photon optomechanical couplings are in the sub-Hz range, whereas single-photon optical nonlinearities ν are in the kHz range. Thus, the regime $\mathcal{C}_{\mathrm{thr}}\ll 1$ is compatible with the bad-cavity limit even for disks with large mechanical quality factors. Moreover, the nonlinear corrections to the Langevin equations (2) will be small.

Our scheme could also find application in feedback cooling based on optomechanical position detection [52]. Cooling down to the mechanical ground state places much more stringent demands on the measurement imprecision than the SQL [8], $\bar{S}_{xx}^{(\mathrm{imp})} \lesssim \bar{S}_{xx}^{\mathrm{SQL}}/\bar{n}^{(\mathrm{th})}$, where $\bar{n}^{(\mathrm{th})}$ is the initial number of thermal phonons. This requires circulating photon numbers enlarged by a factor $\bar{n}^{(\mathrm{th})}$, such that the reduction afforded by our squeezing scheme becomes even more relevant.

In conclusion, we have shown that the precision of optomechanical position detection can be strongly enhanced by deamplification without loss of quantum efficiency in a monolithic on-chip solution. Our method could pave the way for the quantum-limited position detection of mechanical resonators with larger decay rates. This would allow faster detection of forces yielding an increase of the bandwidth of quantum-limited detectors based on optomechanical devices [2]. A natural extension of our scheme could find application in QND qubit state detection.

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