PHYSICAL REVIEW A **85**, 021802(R) (2012)

Optomechanical cooling of levitated spheres with doubly resonant fields

G. A. T. Pender,[1] P. F. Barker,[1] Florian Marquardt,[2,3] J. Millen,[1] and T. S. Monteiro[1,*]
[1]*Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom*
[2]*Institut for Theoretical Physics, Universität Erlangen-Nürnberg, Staudtstraße 7, D-91058 Erlangen Germany*
[3]*Max Planck Institute for the Science of Light, Günther-Scharowsky-Straße 1/Bau 24, 91058 Erlangen Germany*
(Received 13 July 2011; published 3 February 2012)

Optomechanical cooling of levitated dielectric particles represents a promising new approach in the quest to cool small mechanical resonators toward their quantum ground state. We investigate two-mode cooling of levitated nanospheres in a self-trapping regime. We identify a structure of overlapping, multiple cooling resonances and strong cooling even when one mode is blue-detuned. We show that the best regimes occur when both optical fields cooperatively cool and trap the nanosphere, where cooling rates are over an order of magnitude faster compared to corresponding single-resonance cooling rates.

DOI: 10.1103/PhysRevA.85.021802 PACS number(s): 42.50.Wk, 37.30.+i, 37.10.Vz

Extraordinary progress has been made in the past half-dozen years [1,2] toward the final goal of cooling a small mechanical resonator down to its quantum ground state and hence of realizing quantum behavior in a macroscopic system. Implementations include cavity cooling of micromirrors on cantilevers [3–6], dielectric membranes in Fabry-Pérot cavities [7], radial and whispering gallery modes of optical microcavities [8], and nanoelectromechanical systems [9]. Indeed, the realizations span 12 orders of magnitude [2], up to and including the Laser Interferometer Gravitational-Wave Observatory (LIGO) gravity wave experiments. Ground-state cooling has recently been finally attained [10]. Corresponding advances in the theory of optomechanical cooling have also been made [11–14].

Over the past year or so, a promising new paradigm has been attracting much interest: several groups [15–18] have now proposed schemes for optomechanical cooling of levitated dielectric particles, including nanospheres and even viruses [15,19]. The important advantage is the elimination of the mechanical support, a dominant source of heating noise. In general, these proposals involve two fields, one for trapping and one for cooling. This may involve an optical cavity mode plus a separate trap, or two optical cavity modes, the so-called "self-trapping" scenario. Scenarios using two or more cavity modes have also been investigated in nonlevitated systems [20,21].

Mechanical oscillators in the self-trapping regime differ from other optomechanically cooled devices in a second fundamental respect (in addition to the absence of mechanical support): the mechanical frequency, ω_M, associated with center-of-mass oscillations is not an intrinsic feature of the resonator but is determined by the optical field. In particular, it is a function of one or both of the detuning frequencies, δ_1 and δ_2, of the optical modes. Cooling, in general, occurs when ω_M is resonantly red-detuned with either of the detuning frequencies (i.e., negative $\delta_{1,2}$ is associated with cooling). For self-trapping systems, this means $\omega_M(\delta_1,\delta_2) \sim -\delta_{1,2}$ so the relevant frequencies are not independent.

The full implications of this nonlinear interdependence of the resonant frequencies have not yet been fully elucidated. We show here that it leads to a rich landscape of multiple cooling resonances associated with enhanced cooling. The mechanism here is unrelated to splittings seen in experiments in the strong-coupling regime [22]. However, it results in extremely favorable cooling regimes, where up to three cooling resonances can overlap. We investigate in detail the "double-resonance" regime in which two resonances overlap, and we find it can produce cooling rates nearly two orders of magnitude stronger than previously studied "single-resonance" self-trapped cases.

A self-trapping Hamiltonian was investigated in [16] and corresponds to the setup illustrated in Fig. 1:

$$\frac{\hat{H}}{\hbar} = -\delta_1\hat{a}_1^\dagger\hat{a}_1 - \delta_2\hat{a}_2^\dagger\hat{a}_2 + \frac{\hat{P}^2}{2m} - A\hat{a}_2^\dagger\hat{a}_2\cos^2(k_2\hat{x}-\phi) - A\hat{a}_1^\dagger\hat{a}_1\cos^2 k_1\hat{x} + E_1(\hat{a}_1^\dagger + \hat{a}_1) + RE_1(\hat{a}_2^\dagger + \hat{a}_2). \quad (1)$$

Two optical field modes $\hat{a}_{1,2}$ are coupled to a nanosphere with center-of-mass position x. $\hat{H}$ is given in the rotating frame of the laser, which drives the modes with amplitudes E_1 and RE_1, respectively. ϕ is the phase between the optical potentials. The study in [16] investigated the $\delta_1 \simeq 0$ regime, where the $\hat{a}_1$ mode is responsible exclusively for trapping while the $\hat{a}_2$ mode alone provides cooling. Previous studies [15,16,18] analyzed mechanical oscillations about an equilibrium position $x_0 \simeq 0$, corresponding to the antinode of the trapping mode (field 1). Below, this scenario is referred to as the "single-resonance" regime.

Here we investigate the effects of relaxing these restrictions, allowing both fields to simultaneously trap and cool the particle, and we find interesting and unexpected implications. We take $\phi = \pi/4$; one field is driven more weakly than the other, with a ratio $R \simeq 0.1$–1. Below, our analytical expressions cover arbitrary κ, R, E_1, and A, but we compare with an illustrative set of *experimentally plausible parameters*: we take a cavity damping $\kappa = 6\times10^5$ Hz. We considered driving powers in the range $P \simeq 1$–10 mW, where $P = \frac{4kcE_1^2\hbar}{\kappa}$. For a laser of wavelength $\lambda = 1064$ nm and a cavity of length $L \sim 1$ cm and waist 25 μm, we consider a silica nanosphere of ~ 100 nm radius, $m \simeq 9.2\times10^{-18}$ Kg, and hence a coupling strength $A \simeq 3\times10^5$ Hz. The frequency difference between the modes is $|\omega_1-\omega_2| \sim 2\pi\times10$ GHz. This far exceeds the detunings from cavity resonance, $\delta_{1,2} \sim$ 1 MHz, and also the mechanical frequencies ω_M. Thus the

*t.monteiro@ucl.ac.uk

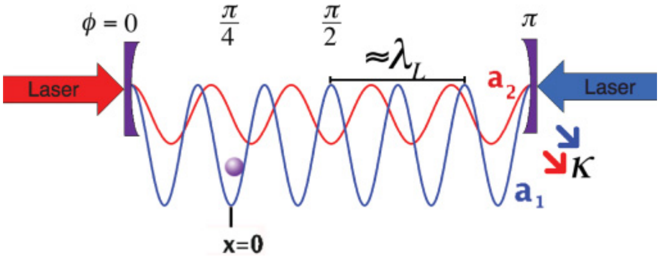

FIG. 1. (Color online) Schematic setup: a levitated nanosphere is trapped and cooled cooperatively by two optical modes. The optical potential for each mode is shown.

photons are completely distinguishable and can be read out and driven separately. Nevertheless, since $\omega_{1,2} \sim 10^{15}$ Hz, we approximate $k_1 \simeq k_2 \equiv k$.

Figure 2 illustrates the behavior for $R = 0.5$. It shows that allowing both fields to cooperatively trap and cool yields more than an additive improvement. We denote by $r1$ and $r2$ the set of detunings corresponding to cooling resonances of modes 1 and 2, respectively: Fig. 2 shows that these resonances unexpectedly split and separate into new cooling resonances $r1\pm$ and $r2\pm$. These can overlap to give very strong cooling associated with multiple resonances. Very strong cooling is apparent even for regimes in which one mode is blue-detuned. In addition, although there is no direct coupling between optical modes, double resonances offer the prospect of strong (albeit second-order) coupling and entangling of the two modes via the nanosphere, within a single cavity. This includes simultaneous resonant and antiresonant regimes (indicated by the crossing of $r2+$ and $a1-$ in Fig. 2) where one mode resonantly heats while the other resonantly cools the mechanical mode.

The dynamics depends on k, m, A, κ, δ_1, δ_2, E_1, and R. However, transforming to scaled variables reduces this complexity. We rescale position, time, and field variables as follows: $kx \to \tilde{x}$, $At \to \tilde{t}$, then $a_{1,2} \to \frac{E_1}{iA}\tilde{a}_{1,2}$. Note that below, we drop all the tildes but it is implicit that all variables are scaled in the resulting Heisenberg equations:

$$\begin{aligned} \ddot{\hat{x}} &= -\epsilon^2[|\hat{a}_1|^2 \sin 2\hat{x} + |\hat{a}_2|^2 \sin(2\hat{x} - \pi/2)], \\ \dot{\hat{a}}_1 &= i\Delta_1\hat{a}_1 + 1 + i\hat{a}_1 \cos^2 \hat{x} - \kappa_A\hat{a}_1, \\ \dot{\hat{a}}_2 &= i\Delta_2\hat{a}_2 + R + i\hat{a}_2 \cos^2(\hat{x} - \pi/4) - \kappa_A\hat{a}_2. \end{aligned} \tag{2}$$

The dynamics for a given $R < 1$ depends only on the scaled driving $\epsilon^2 = \zeta E_1^2$, where $\zeta = \frac{\hbar k^2}{mA^3}$, two scaled detunings $\Delta_{1,2} = \delta_{1,2}/A$, and a scaled damping $\kappa_A = \frac{(\kappa/2)}{A}$; all scaled frequencies (including cooling rates) are given below as a fraction of A.

The experimentally adjustable parameters are ϵ, both the detunings $\Delta_{1,2}$, and R. We assume $\kappa_A \simeq 1$, though the analytical expressions are for arbitrary κ_A. Varying driving power $P \sim 1$–10 mW but leaving the cavity and nanosphere properties unchanged means A remains constant but ϵ^2 varies from $\sim$1 to 100.

We may analyze the cooling classically by replacing operators by their expectation values and linearizing about equilibrium fields, introducing the shifts $a_1 \to \alpha_1 + a_1$, $a_2 \to \alpha_2 + a_2$, and $x \to x_0 + x$. Hence we find equilibrium photon

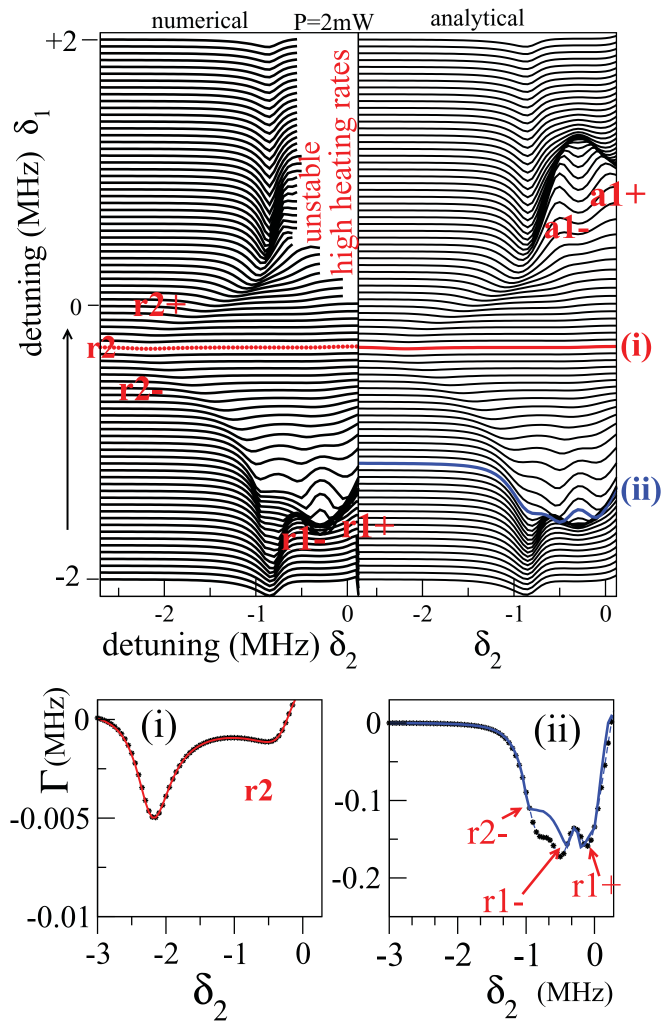

FIG. 2. (Color online) Upper panels: comparison between numerical optical cooling rates (without linearization) and an analytical expression [Eq. (5)] from linearized dynamics, showing excellent agreement. $R = 0.5$. The curves corresponding to different values of $\delta_1 \pm 2$ MHz are shifted relative to each other. At single-resonance $r2$, field 2 is resonant with the oscillator and is exclusively responsible for cooling; field 1 is resonant with the cavity ($\Delta_1^x = 0$) and traps the sphere. Subsequently $r2\pm$ appear: they are cooling resonances of field 2, split by field 1; conversely, $r1\pm$ are cooling resonances of field 1, split by field 2. $r2-$,$r1-$,$r1+$ overlap giving a broad region of very strong cooling. The $a1\pm$ are heating (Stokes) resonances of field 1. The $a1-$ can coincide with the cooling resonance ($r2+$). Here field 2 absorbs phonons as fast as field 1 emits them. Lower panels: unshifted cooling curves at the single-field cooling resonance $r2$ (i) and double-resonant cooling (ii), showing that the latter gives over an order of magnitude stronger cooling. Asterisks are numerical results and the curves analytical results, corresponding to curves marked (i) and (ii), respectively, in the upper panels.

fields, $\alpha_1 = [\kappa_A - i\Delta_1^x]^{-1}$ and $\alpha_2 = R[\kappa_A - i\Delta_2^x]^{-1}$, as well as position $\tan 2x_0 = |\alpha_2|^2/|\alpha_1|^2$.

Here, $\Delta_1^x = \Delta_1 + \frac{1}{2}(1 + \cos 2x_0)$ and $\Delta_2^x = \Delta_2 + \frac{1}{2}(1 + \sin 2x_0)$. The dimensionless mechanical frequency is

$$\omega_M^2(\Delta_1, \Delta_2) = 2\epsilon^2(|\alpha_1|^2 \cos 2x_0 + |\alpha_2|^2 \sin 2x_0). \tag{3}$$

Closely related forms of this "self-trapping" frequency expression have been noted previously [15–18], but the implications, other than for $x_0 \simeq 0$, have not been investigated.

To first order, the linearized equations of motion are

$$\begin{aligned}\ddot{x} &= -\omega_M^2 x - \epsilon^2(g_1 \sin 2x_0 - g_2 \cos 2x_0),\\ \dot{a}_1 &= i\Delta_1^x a_1 - i\alpha_1 x \sin 2x_0 - \kappa_A a_1,\\ \dot{a}_2 &= i\Delta_2^x a_2 + i\alpha_2 x \cos 2x_0 - \kappa_A a_2,\end{aligned} \tag{4}$$

where $g_i = (\alpha_i^* a_i + \alpha_i a_i^*)$. From the above, we can obtain the contribution from the two photon fields to the optomechanical cooling:

$$\frac{\Gamma}{2} = \Omega\,[S_1(\omega_M) + S_2(\omega_M) - S_1(-\omega_M) - S_2(-\omega_M)], \tag{5}$$

where $\Omega = \frac{\epsilon^2\kappa_A}{2\omega_M}$ with $S_1(\omega) = \frac{|\alpha_1|^2 \sin^2 2x_0}{[\Delta_1^x-\omega]^2+\kappa_A^2}$ and $S_2(\omega) = \frac{|\alpha_2|^2 \cos^2 2x_0}{[\Delta_2^x-\omega]^2+\kappa_A^2}$. Note that net cooling occurs for $\Gamma < 0$. We also calculate a numerical Γ by evolving the equations of motion in time and looking at the decay in $x(t)$ (its variance in particular). The analytical cooling rates give excellent agreement with numerics in all but the strongest cooling regions.

The single-field cooling resonance $r2$ occurs for $x_0 \simeq 0$, $\Delta_1^x \simeq 0$, and $\Delta_2^x = -\omega_M$, thus here for $\Delta_1 = -1$ (i.e., $\delta_1 = -A$). Conversely, there is also a single-field cooling resonance $r1$ for $x_0 = \pi/4$. Away from these extreme cases, both cooling resonances are split by the effect of the other field, wherever $0 < x_0 < \pi/4$. Note that we need only consider the case $R \leqslant 1$, i.e., field 2 is always driven more weakly than field 1. The range $R > 1$ represents simply an interchange in the role of the fields: unlike previous studies, we no longer have a distinct trapping field and a cooling field.

From Fig. 2, we see $r2\pm$ occur for the same Δ_2, thus the same equilibrium photon field α_2; however, they correspond to photon fields α_1 and α_1^*, respectively, and thus to different $\Delta_1^{\pm} = \pm y_1$, where $2y_1$ is the splitting (about $\Delta_1^x = 0$) between $r2+$ and $r2-$ seen in Fig. 2.

The transformation $\alpha_1 \to \alpha_1^*$ leaves both the mechanical frequency $\omega_m(\Delta_1^{\pm},\Delta_2)$ and $x_0(\Delta_1^{\pm},\Delta_2)$ unchanged. Hence the cooling rates are similar for both $r2\pm$.

We can estimate the splitting $\Delta_1^{\pm}$ by requiring $\omega_M(\Delta_1^+,\Delta_2) = \omega_M(\Delta_1^-,\Delta_2) \simeq \Delta_2^x$ since $\omega_M(\Delta_1^{\pm},\Delta_2) \simeq \Delta_2^x$ are the conditions for the optomechanical resonance $r2\pm$. Thus we obtain $y_1 = \sqrt{\frac{2\epsilon^2}{(\Delta_2^x)^2 \cos 2x_0} - \kappa_A^2}$. Close to $r2$, we can simplify $y_1 \simeq \frac{\sqrt{2}\epsilon}{\Delta_2 + A/2}$. Similarly, y_2, the splitting between $r1\pm$, is $y_2 = \sqrt{\frac{2\epsilon^2 R^2}{(\Delta_1^x)^2 \sin 2x_0} - \kappa_A^2}$.

Thus the splittings increase with driving power and R. Though the behavior is reminiscent of optical bistabilities, we stress that the splitting here is of the cooling resonances. In Fig. 3, the deep cooling region due to the three overlapping resonances $r1\pm, r2-$ is shown for scaled parameters, thus this is generic behavior not due to any particular value of A, E_1, or m but a function only of the subset of scaled variables.

We now analyze the relative merits of single-resonance versus double-resonance cooling. Single-field cooling corresponds to $r2$ in Fig. 2. Cooling rates are obtained from Eq. (5) by taking $x_0 \simeq 0$, $\Delta_1^x = 0$, and $\Delta_2^x = -\omega_M$. Further, we can approximately estimate cooling rates purely in terms of experimental parameters (driving power, R, and κ). Assuming $S_2(-\omega_M) \gg S_2(+\omega_M)$ and that the field 1 contribution to cooling is negligible, near $r2$, the mechanical frequency $\omega_M^2 = \frac{2\epsilon^2}{\kappa_A^2}$. Hence, the single-resonance (SR) cooling rate becomes

$$-\Gamma_{SR} \approx \left(R^2\epsilon\kappa_A^2\right)\left(2\sqrt{2}\epsilon^2 + \sqrt{2}\kappa_A^4\right)^{-1} \tag{6}$$

(recall this is a scaled cooling rate thus given in units of A). Single-field cooling is a maximum if $\epsilon = \kappa_A^2/\sqrt{2}$, where

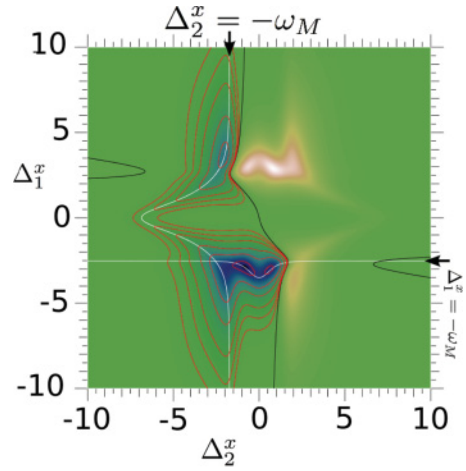

FIG. 3. (Color online) Dimensionless cooling rates Γ as a function of scaled (dimensionless) detunings for $\epsilon^2 = 24$, $\frac{\kappa}{2A} = 1$, and $R = 1/2$. The inner contour corresponds to $\Gamma = 1/2$ (where $\Gamma = 1$ would correspond to a maximal cooling rate $= \kappa/2$). The outer contour indicates $\Gamma = 1/80$, with intermediate contours at $\Gamma = 1/4$, 1/8, 1/20, and 1/40, respectively. The black contour denotes $\Gamma = 0$ and indicates the boundary between cooling and heating.

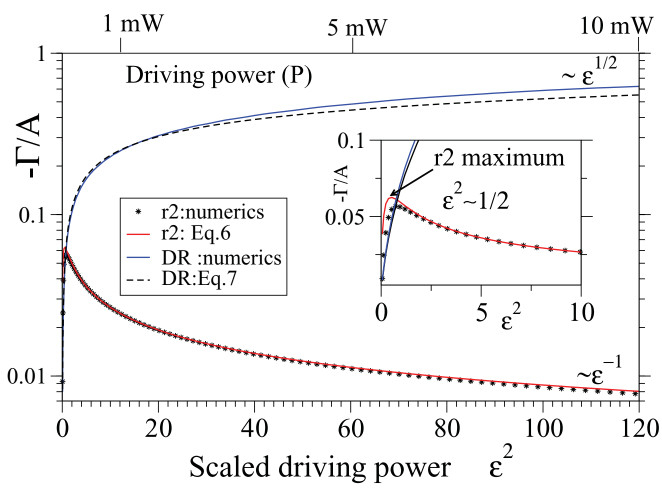

FIG. 4. (Color online) Comparison between single-resonant (SR) cooling due to $r2$ and double resonant (DR) cooling, i.e., simultaneous cooling by $r1-$ and $r2-$, as a function of laser driving power. Double resonance yields two orders of magnitude greater cooling for strong driving. The inset shows a cooling maximum of $r2$ at weak driving $\epsilon^2 \simeq 1/2$; here the system is on the edge of the sideband-resolved regime $\omega_M = \kappa_A$. The upper axes give input laser powers corresponding to $A = 3 \times 10^5$ Hz$= \kappa/2$, $m = 9 \times 10^{-18}$ Kg, and $R = 0.5$.

$\omega_M = \kappa/2$ (in unscaled units), and is thus at the edge of the resolved sideband regime. Here, $-\Gamma_{SR} \approx \frac{R^2}{4}$; this gives optimal cooling $\Gamma \sim \kappa$ only if $R \sim 1$. This cooling maximum is independent of κ_A: it depends only on R. As the driving is increased, if $2\epsilon^2 \gg \kappa_A^4$, then $-\Gamma_{SR} \sim \frac{R^2\kappa_A^2}{2\sqrt{2}\epsilon} \propto 1/\epsilon$. Thus the single-resonance cooling rate falls off quite rapidly as the driving amplitude is increased: the cooling cannot be improved by increasing the driving amplitude.

To obtain the corresponding double-resonant rate (DR), one must first identify the ϵ-dependent pair of detunings for which $-\omega_M(\Delta_1,\Delta_2) \approx \Delta_1^x \approx \Delta_2^x$. Even if $r1-$, $r2-$ do not cross in Fig. 2, the sidebands approach within their width κ and overlap significantly. One can still obtain a good approximation to the cooling rate in terms of driving parameters. In this case, there are contributions to cooling from both field 1 and field 2. Adding them both,

$$-\Gamma_{DR} \approx \epsilon^2(R^2+R^4)\left[\kappa_A\omega_M\left(\omega_M^2+\kappa_A^2\right)\right]^{-1}, \qquad (7)$$

where the frequency is given by the expression $\omega_M^2 = \frac{-\kappa_A^2}{2} + \frac{1}{2}\sqrt{\kappa_A^4 + 8\epsilon^2}$. The contribution from mode 2 $\propto R^2$ while that of mode 1 $\propto R^4$; both contribute significantly for $R \gtrsim 0.5$. Assuming $\omega_M \gg \kappa_A$, this reduces to $-\Gamma_{DR} \approx 2^{-3/4}(R^2+R^4)\epsilon^{1/2}/\kappa_A$, hence $\Gamma_{DR} \propto \epsilon^{1/2}$. Figure 4 shows that Eqs. (6) and (7) both give excellent agreement with exact numerics. In the double-resonant case, the cooling is stronger and increases with increasing ϵ. In contrast, the single-resonant cooling $\Gamma_{SR} \propto 1/\epsilon$ and this cannot be improved by increasing ϵ. Self-trapping cooling cannot be considered simply in terms of an additive contribution from two intracavity intensities; the response of ω_M to the driving is also important. In the double-resonant case, $\omega_M \propto \epsilon^{1/2}$ for strong driving. In contrast, for the single-resonant case $\omega_M \propto \epsilon$ and strong driving pushes the $r2$ resonance into the far-detuned regime.

A study of the quantum cooling shows that, in the absence of mechanical damping, the minimum phonon numbers attainable are $\bar{n}_{min} \ll 1$ for both single and double-resonant cooling, provided the driving is strong, i.e. if $\epsilon^2 \gg 1$. However, a current key technical barrier to the first actual realization of cavity-cooled levitated oscillators is in pumping down to a sufficiently high vacuum (pressure $P \sim 10^{-6}$ mBar) to ensure the mechanical damping rate $\gamma_M \propto P$ is negligible without losing an untethered levitated object. A detailed study of Langevin-Heisenberg equations in our system, including gas collisions, confirms previous estimates [15] that for typical regimes with $\Gamma \gg \gamma_M$, the final equilibrium temperature $T_{eq} \approx T_g \frac{\gamma_M}{|\Gamma|}$, where T_g is the gas temperature. Thus, as seen in Fig. 4, for 5 mW, for example, the DR regime will attain ground-state cooling at a pressure about 40 times higher than the SR regime.

In conclusion, our study shows that the two-mode self-trapping regime reveals a range of unexpected features, including the double and triple cooling resonances and strong cooling at blue-detuning. Although other proposals also permit strong cooling rates, the multiple sidebands provide an exceptionally broad region of strong cooling.

G.A.T.P., P.F.B., and T.M. acknowledge support from the EPSRC. F.M. thanks the DFG (Emmy-Noether grant), DARPA ORCHID and an ERC Starter grant.

[1] F. Marquardt and S. Girvin, Physics **2**, 40 (2009).
[2] T. Kippenberg and K. Vahala, Science **321**, 1172 (2008).
[3] C. H. Metzger and K. Karrai, Nature (London) **432**, 1002 (2004).
[4] O. Arcizet *et al.*, Nature (London) **444**, 71 (2006).
[5] S. Gigan *et al.*, Nature (London) **444**, 67 (2006).
[6] C. A. Regal *et al.*, Nat. Phys. **4**, 555 (2008).
[7] J. D. Thompson *et al.*, Nature (London) **452**, 72 (2008); A. M. Jayich *et al.*, New J. Phys **10**, 095008 (2008).
[8] A. Schliesser *et al.*, Nat. Phys. **5**, 509 (2009).
[9] A. Naik *et al.*, Nature (London) **443**, 193 (2006); A. D. Armour, M. P. Blencowe, and K. C. Schwab, Phys. Rev. Lett. **88**, 148301 (2002).
[10] J. D. Teufel *et al.*, Nature (London) **475**, 359 (2011); J. Chan *et al.*, *ibid.* **478**, 89 (2011).
[11] V. B. Braginski and F. Y. Khalili *Quantum Measurement* (Cambridge University Press, Cambridge, UK, 1992).
[12] M. Paternostro *et al.*, New J. Phys. **8**, 107 (2006).
[13] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, Phys. Rev. Lett. **99**, 093902 (2007).
[14] I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, Phys. Rev. Lett. **99**, 093901 (2007).
[15] O. Romero-Isart, M. L. Juan, R. Quidant, and J. I. Cirac, New J. Phys. **12**, 033015 (2010).
[16] D. E. Chang, C. A. Regal, S. B. Papp, D. J. Wilson, O. Painter, H. J. Kimble, and P. Zoller, Proc. Natl. Acad. Sci. (USA) **107**, 1005 (2010).
[17] P. F. Barker and M. N. Shneider, Phys. Rev. A **81**, 023826 (2010).
[18] R. J. Schulze, C. Genes, and H. Ritsch, Phys. Rev. A **81**, 063820 (2010).
[19] O. Romero-Isart *et al.*, Phys. Rev. A **83**, 013803 (2011).
[20] T. Corbitt *et al.*, Phys. Rev. Lett. **98**, 150802 (2007); H. Miao *et al.*, Phys. Rev. A **78**, 063809 (2008); C. Zhao *et al.*, Phys. Rev. Lett. **102**, 243902 (2009).
[21] J. M. Dobrindt and T. J. Kippenberg, Phys. Rev. Lett. **104**, 033901 (2010).
[22] S. Groblacher *et al.*, Nature (London) **460**, 724 (2009).